

Steam nozzles

(1)

Nozzle:-

It is a device which continuously varying cross-section & increasing K.E by conserving the pressure energy.

Applications of nozzles:-

- fire engines / water services
- Kinetic energy increases
- spray pump
- Steam jet injectors
- I.C engine

Types of nozzles:-

According to the way of varying cross-section

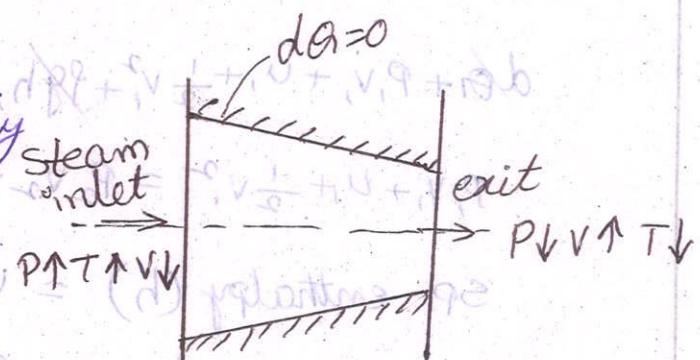
i) Convergent nozzle

ii) Divergent nozzle

iii) Convergent-divergent nozzle

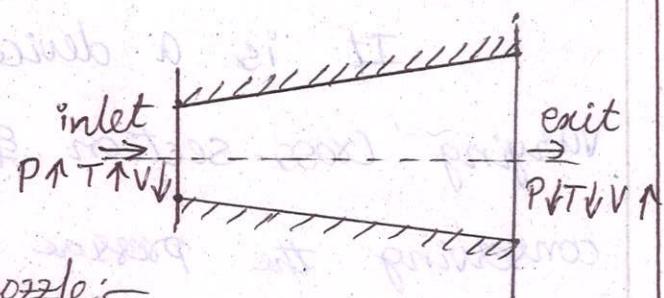
iv) Convergent nozzle:-

It continuously decreasing cross-section.

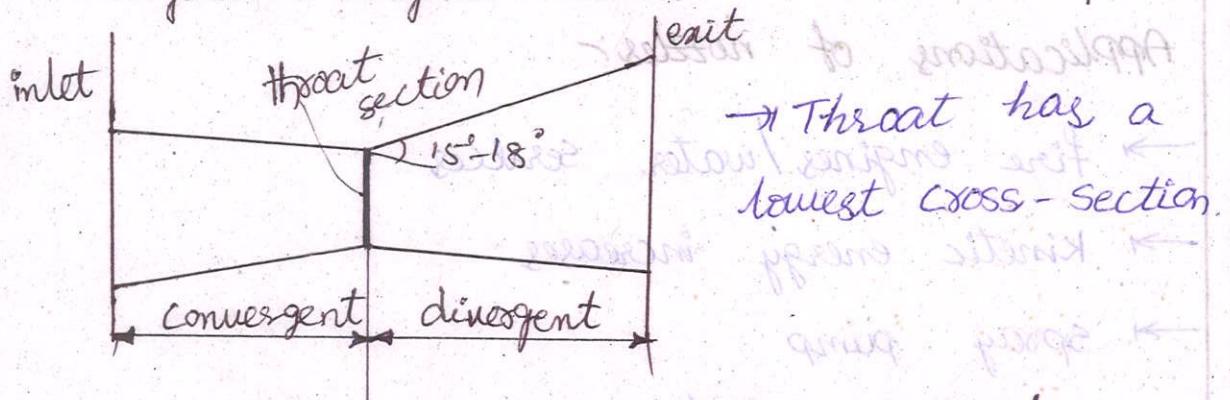


ii) Divergent nozzle:

It increasing cross-section.



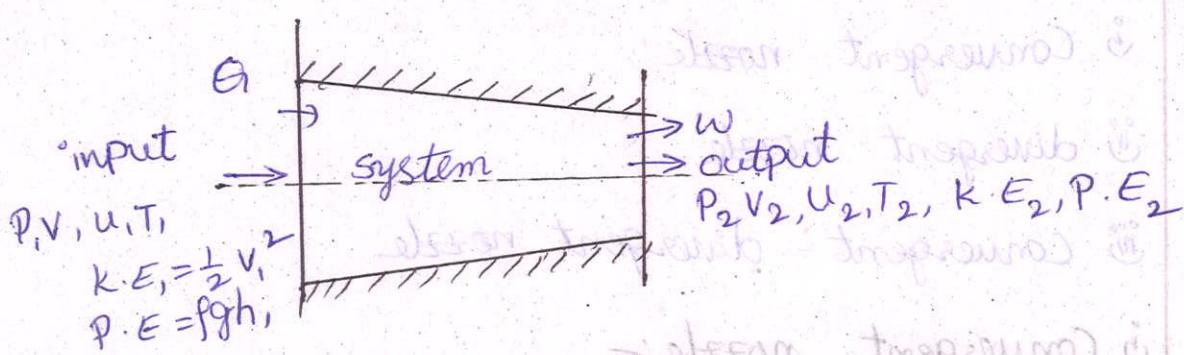
iii) Convergent-divergent nozzle:



Thermodynamic analysis of steam nozzle:

Consider SFEE (steady flow energy eqⁿ)

$$E_i/p = E_o/p$$



$$d\phi_1 + P_1 V_1 + U_1 + \frac{1}{2} V_1^2 + fgh_1 = W + P_2 V_2 + U_2 + \frac{1}{2} V_2^2 + fgh_2$$

$$P_1 V_1 + U_1 + \frac{1}{2} V_1^2 = P_2 V_2 + U_2 + \frac{1}{2} V_2^2$$

$$\text{sp. enthalpy } (h) = U + PV$$

$$h_1 + \frac{1}{2} V_1^2 = h_2 + \frac{1}{2} V_2^2$$

$$V_2 \gg V_1$$

$$h_1 = h_2 + \frac{1}{2} V_2^2$$

V_2 = exit velocity of nozzle

$$h_1 - h_2 = \frac{1}{2} V_2^2$$

$$V_2^2 = 2(h_1 - h_2)$$

$$V_2 = \sqrt{2(h_1 - h_2)}$$

$$V_2 = \sqrt{2(1000h_1 - 1000h_2)}$$

$$= \sqrt{2000(h_1 - h_2)}$$

$$V_2 = 44.72 \sqrt{h_1 - h_2}$$

Mass flow rate in a nozzle:-

mass flow rate with respect to time.

mass = acceleration due to gravity.

$$\text{mass density } (\rho) = \frac{\text{mass } (m)}{\text{volume } (V)}$$

$$\frac{m}{t} = \frac{\rho V}{t}$$

\dot{m} = mass flow rate

$$\dot{m} = \frac{\rho V}{t} = \frac{\rho A \cdot L}{t}$$

$$\dot{m} = \rho \cdot A \left(\frac{L}{t} \right)$$

$$\dot{m} = \rho \cdot A \cdot V$$

$$\dot{m} = \frac{AV}{U}$$

$$\frac{1}{\rho} = \frac{1}{\text{density}} = \left[\frac{(V_2)}{V} - 1 \right] \frac{V_2}{V} \quad (V = \text{exit velocity, nozzle})$$

$$\dot{m} = (\rho AV) \left[\left(\frac{V_2}{V} \right) - 1 \right] \left(\frac{V_2}{V} \right)$$

where; ρ = density of steam

A = A/c of nozzle

$$= \frac{AV_2}{(1/\rho)}$$

$$\dot{m} = \frac{AV_2}{\nu}$$

$$\frac{V_2^2}{2} = \frac{n}{n-1} (P_1 V_1 - P_2 V_2)$$

$$V_2 = \sqrt{\left(\frac{2n}{n-1}\right) (P_1 V_1 - P_2 V_2)}$$

$$V_2 = 44.72 \sqrt{h_1 - h_2}$$

$$\dot{m} = \frac{A}{\nu} \sqrt{\left(\frac{2n}{n-1}\right) (P_1 V_1 - P_2 V_2)}$$

$$\dot{m} = \frac{A}{\nu} \sqrt{\left(\frac{2n}{n-1}\right) P_1 V_1 \left[1 - \left(\frac{P_2 V_2}{P_1 V_1}\right)\right]}$$

Polytropic process $PV^n = C$

$$P_1 V_1^n = P_2 V_2^n$$

$$\left(\frac{V_1}{V_2}\right)^n = \left(\frac{P_2}{P_1}\right)$$

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\frac{V_2}{V_1} = \left(\frac{P_2}{P_1}\right)^{-1/n}$$

$\frac{V_2}{V_1}$ sub. in \dot{m} eqn

$$\dot{m} = \frac{A_2}{\nu_2} \left(\frac{2n}{n-1}\right) P_1 V_1 \left[1 - \left(\frac{P_2 V_2}{P_1 V_1}\right)\right]$$

$$= \frac{A_2}{\nu_2} \sqrt{\left(\frac{2n}{n-1}\right) \left[P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right) \left(\frac{P_2}{P_1}\right)^{-1/n}\right]\right]}$$

(3)

$$\dot{m} = \frac{A_2}{U_2} \left[\frac{(2n)}{(n-1)} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \right]$$

The process in nozzle is polytropic process.

$$P_1 V_1^n = P_2 U_2^n$$

$$\frac{P_1 V_1^n}{P_2} = U_2^n$$

$$\left[\left(\frac{P_1}{P_2} \right) (V_1)^n \right]^{1/n} = U_2$$

$$U_2 = V_1 \left(\frac{P_1}{P_2} \right)^{1/n}$$

$$\dot{m} = \frac{A_2}{\left[V_1 \left(\frac{P_1}{P_2} \right)^{1/n} \right]} \left[\frac{(2n)}{(n-1)} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \right]$$

$$= A_2 \left[\frac{(2n)}{(n-1)} \frac{P_1 V_1}{\left[V_1 \left(\frac{P_1}{P_2} \right)^{1/n} \right]^2} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \right]$$

$$= A_2 \left[\frac{(2n)}{(n-1)} \frac{P_1 V_1}{V_1^2 \left(\frac{P_1}{P_2} \right)^{2/n}} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \right]$$

$$= A_2 \left[\frac{(2n)}{(n-1)} \left(\frac{P_1}{U_1} \right) \left[\frac{1}{\left(\frac{P_1}{P_2} \right)^{2/n}} - \frac{\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}}{\left(\frac{P_1}{P_2} \right)^{2/n}} \right] \right]$$

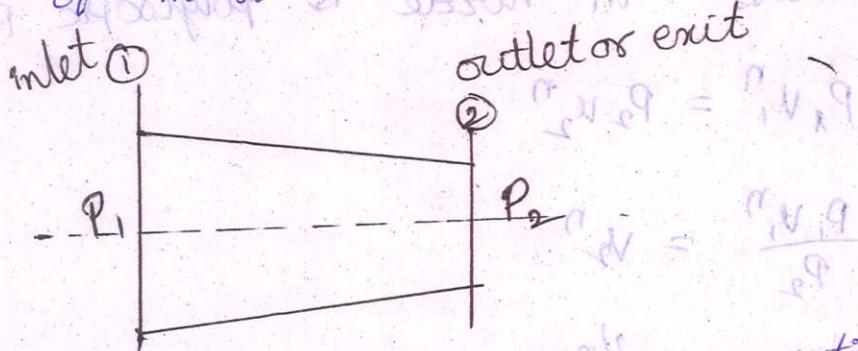
$$= A_2 \left[\frac{(2n)}{(n-1)} \frac{P_1}{U_1} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n} + \frac{2}{n}} \right] \right]$$

$$\dot{m} = A_2 \left[\frac{(2n)}{(n-1)} \frac{P_1}{U_1} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right] \right]$$

Critical Pressure ratio of nozzle:

At the critical pr. ratio the \dot{m}_{max} .

amount of mass flow is possible.



At the critical pressure ratio the max. amount of mass flow is possible.

$$\dot{m} = A_2 \left[\frac{(2n)}{n-1} \frac{P_1}{u_1} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{n+1/n} \right] \right]$$

$$\frac{d\dot{m}}{d \left(\frac{P_2}{P_1} \right)} = 0$$

$$\frac{d\dot{m}}{d \left(\frac{P_2}{P_1} \right)} = \frac{d}{d \left(\frac{P_2}{P_1} \right)} \left[A_2 \left[\frac{(2n)}{n-1} \frac{P_1}{u_1} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{n+1/n} \right] \right] \right] = 0$$

$$\frac{d}{dx} \sqrt{f(x)} = \frac{1}{\sqrt{f(x)}} \frac{d}{dx} [f(x)] \quad \left[\text{if } \frac{d}{dx} x^n = nx^{n-1} \right]$$

$$\frac{d\dot{m}}{d \left(\frac{P_2}{P_1} \right)} = \frac{1}{2} \left[\frac{(2n)}{n-1} \frac{P_1}{u_1} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{n+1/n} \right] \right]$$

$$\frac{d}{d \left(\frac{P_2}{P_1} \right)} \left[\frac{(2n)}{n-1} \frac{P_1}{u_1} \left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{n+1/n} \right] = 0$$

$$\Rightarrow \left(\frac{2n}{n-1} \right) \frac{P_1}{u_1} \frac{d}{d \left(\frac{P_2}{P_1} \right)} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{n+1/n} \right] = 0$$

$$\frac{d}{d\left(\frac{P_2}{P_1}\right)} \left[\left(\frac{P_2}{P_1}\right)^{2/n} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}} \right] = 0$$

$$\frac{d}{d\left(\frac{P_2}{P_1}\right)} \left(\frac{P_2}{P_1}\right)^{2/n} - \frac{d}{d\left(\frac{P_2}{P_1}\right)} \left[\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right] = 0$$

$$\frac{2}{n} \left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}} - \left[\left(\frac{n+1}{n}\right) \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}-1}\right] = 0$$

$$\frac{2}{n} \left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}} = \left(\frac{n+1}{n}\right) \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$2 \left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}} = (n+1) \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\frac{2}{n+1} = \left(\frac{P_2}{P_1}\right)^{1/n} \left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}}$$

$$= \left(\frac{P_2}{P_1}\right)^{1/n} - \frac{2}{n+1}$$

$$= \left(\frac{P_2}{P_1}\right)^{\frac{1-2+n}{n}}$$

$$\left(\frac{2}{n+1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow \left(\frac{P_2}{P_1}\right) = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

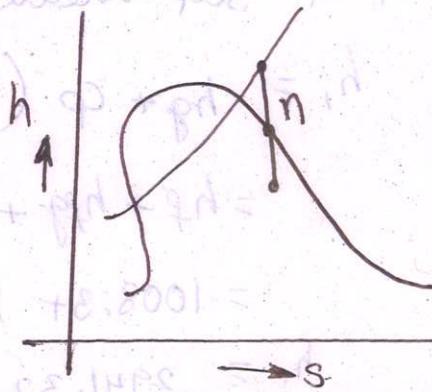
$$\therefore \text{critical pr. ratio} = \boxed{\left(\frac{P_2}{P_1}\right) = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}}$$

'M' value in different region :-

superheated region, $n = 1.3$

wet region, $n = 1.035 + 0.12$

dry saturation, $n = 1.135$



Nozzle with friction:

$$V_2 = 44.72 \sqrt{k(h_1 - h_2)} \quad [k = \text{co-efficient of friction}]$$

Steam at 30 bar & 300°C expands to 5 bar through a nozzle isentropically. calculate exit velocity.

Given data:

$$\text{pressure, } P_1 = 30 \text{ bar}$$

$$\text{temp., } T_1 = 300^\circ\text{C} = 300 + 273 = 573 \text{ K}$$

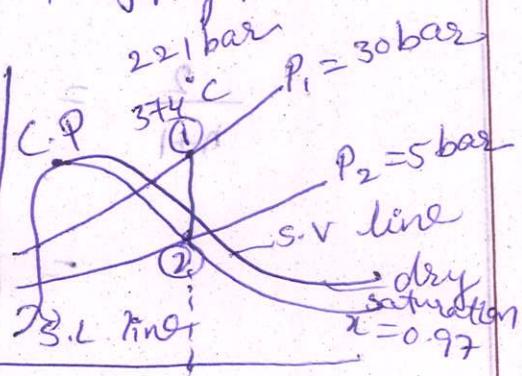
$$\text{pressure, } P_2 = 5 \text{ bar} \quad \therefore C_p = 2.1 \text{ kJ/kg.K}$$

exit velocity of nozzle;

$$V_2 = 44.72 \sqrt{h_1 - h_2}$$

state ① in superheated

state ② in wet-region



by using steam table at 30 bar

$$T_{sat} = 233.8^\circ\text{C}$$

$$h_f = 1008.3 \text{ kJ/kg}, h_{fg} = 1794.0 \text{ kJ/kg}$$

$$s_f = 2.646 \text{ kJ/kg.K}, s_{fg} = 3.538 \text{ kJ/kg.K}$$

h_1 in superheated region;

$$h_1 = h_f + C_p (T_{sup} - T_{sat})$$

$$= h_f + h_{fg} + C_p (T_{sup} - T_{sat})$$

$$= 1008.3 + 1794.0 + 2.1 (300 - 233.8)$$

$$h_1 = 2941.32 \text{ kJ/kg}$$

state (2) in wet region,

$$h_2 = h_f + x_2 \cdot h_{fg}$$

The given process is Isentropic

$$s_1 = s_2$$

$$s_g + c_p \ln\left(\frac{T_{sup}}{T_{sat}}\right) = s_{f_2} + x_2 \cdot s_{fg_2}$$

$$s_f + s_{fg} + c_p \ln\left(\frac{T_{sup}}{T_{sat}}\right) = s_{f_2} + x_2 \cdot s_{fg_2}$$

$$2.646 + 3.538 + 2.1 \ln\left(\frac{300}{233.8}\right) = 1.860 + x_2 \times 4.959$$

$$6.707 = 1.860 + 4.959 x_2$$

$$4.959 x_2 = 6.707 - 1.860$$

$$4.959 x_2 = 4.847$$

$$x_2 = \frac{4.847}{4.959}$$

$$x_2 = 0.97$$

By using steam tables at 5 bar,

$$h_f = 640.1 \text{ kJ/kg}$$

$$h_{fg} = 2107.4 \text{ kJ/kg}$$

$$s_f = 1.860 \text{ kJ/kg K}$$

$$s_{fg} = 4.959 \text{ kJ/kg K}$$

$$h_2 = h_f + x_2 \cdot h_{fg}$$

$$= 640.1 + 0.97 \times 2107.4$$

$$h_2 = 2684.278 \text{ kJ/kg}$$

exit velocity of nozzle

$$V_2 = \sqrt{2g \cdot (h_1 - h_2)}$$

$$= 44.7 \sqrt{2941.32 - 2684.278} \text{ mm } \text{ in } \text{ state}$$

$$V_2 = 716.65 \text{ m/s}$$

for the above problem calculate the area at

exit of the nozzle for flow of 2 kg/sec .

$$\rho = \frac{m}{V}$$

$$\dot{m} = \rho \cdot V$$

$$\dot{m} = \rho \cdot \frac{V}{t} = \rho \cdot A \left(\frac{S}{t} \right) = \rho A V_2$$

$$\dot{m} = \frac{A V_2}{t}$$

by using steam table at 5 bar; $V_g = 0.37466 \text{ m}^3/\text{kg}$

$$V_2 = x_2 V_g$$

$$V_2 = 0.97 \times 0.37466$$

$$= 0.36 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A V_2}{t} \text{ to get exit nozzle from jet}$$

$$A = \frac{\dot{m} V_2}{V_2}$$

$$= \frac{2 \times 0.36}{716.65}$$

$$A = 1.0046 \times 10^{-3} \text{ m}^2$$

$$A = 1.0046 \times 10^{-3} \times (10^2)^2 \text{ cm}^2$$

$$A = 10.046 \text{ cm}^2$$

$$\frac{\pi}{4} d_e^2 = 10.046$$

$$d_e = \sqrt{\frac{10.046 \times 4}{\pi}} \text{ to get exit nozzle diameter}$$

$$d_e = 3.57 \text{ cm}$$

Steam expands isentropically through a nozzle from 15 bar to 490 kPa with initial dry saturation steam. Calculate the exit velocity, neglect the initial velocity. If 10% of heat is lost in friction calculate the % reduction in velocity & improvement in the final quality of the steam.

Given data;

$$P_1 = 15 \text{ bar}$$

$$P_2 = 490 \text{ kPa} \\ = 490 \times 10^3 \text{ Pa}$$

$$10^5 \text{ Pa} = 1 \text{ bar}$$

$$= 1.9 \times 10^2 \times 10^3 \text{ Pa}$$

$$P_2 = 1.9 \text{ bar}$$

By using steam tables at 15 bar;

$$h_f = 844.6 \text{ kJ/kg} ; s_f = 2.314 \text{ kJ/kg K}$$

$$h_{fg} = 1945.3 \text{ kJ/kg} ; s_{fg} = 1.127 \text{ kJ/kg K}$$

at 1.9 bar;

$$h_f = 636.8 \text{ kJ/kg} ; s_f = 1.853 \text{ kJ/kg K}$$

$$h_{fg} = 2109.8 \text{ kJ/kg} ; s_{fg} = 1.973 \text{ kJ/kg K}$$

exit velocity of nozzle;

$$V_2 = 441.72 \sqrt{h_1 - h_2}$$

state ① at the line of dry saturation,

$$x = 1$$

$$h_1 = h_f + x_1 h_{fg}$$

$$h_1 = 844.6 + 1 \times 1945.3$$

$$h_1 = 2789.5 \text{ kJ/kg}$$

state ② wet region:

$$h_2 = h_f + x_2 h_{fg}$$

$$= 636.8 + x_2 (2109.8)$$

Given data steam expands isentropically so,

$$s_1 = s_2$$

$$s_f + x_1 s_{fg} = s_f + x_2 s_{fg}$$

$$2.314 \times 1 \times 4.127 = 1.853 + x_2 \cdot 4.973$$

$$x_2 = 0.92$$

$$h_2 = h_f + x_2 h_{fg}$$

$$= 636.8 + 0.92 (2109.8)$$

$$h_2 = 2577.81 \text{ kJ/kg}$$

exit velocity at nozzle;

$$V_2 = H.H.72 \sqrt{(h_1 - h_2)}$$

$$= 44.72 \sqrt{2789.9 - 2577.81}$$

$$V_2 = 651.27 \text{ m/s}$$

If the 10% of heat lost in the expansion

$$100\% - 10\% = 90\% \text{ So, } K = 0.9$$

$$V_2 = 44.72 \sqrt{K(h_1 - h_2)}$$

$$= 44.72 \sqrt{0.9(2789.9 - 2577.81)}$$

$$V_2 = 617.28 \text{ m/s}$$

% of reduction in velocity in nozzle due to friction

$$\% \text{ Reduction} = \left[\frac{V_{\text{Ideal}} - V_{\text{act}}}{V_{\text{Ideal}}} \right] \times 100$$

$$= \left[\frac{651.27 - 617.28}{651.27} \right] \times 100$$

Element loss = 5.1%, wall roughness loss = 0.1%

Improvement in the final condition of steam

after heat lost; $h_1 - h_2 = 211.5$

$$h_1 - h_2 = 211.5 \quad \} \text{ diff. is } 20\%$$

$$\frac{90}{100} (h_1 - h_2) = 190.5$$

$$\text{Improvement} = 0.92 - 0.912$$

$$h_1 - h_2 = \frac{90}{100} (h_1 - h_2)$$

$$h_1 - h_2 = 0.9h_1 - 0.9h_2'$$

$$h_1 - 0.9h_1 = h_2 - 0.9h_2'$$

$$2789.5 - 0.9 \times 2789.5 = 2577.81 - 0.9h_2'$$

$$0.1h_1 - h_2 = -0.9h_2'$$

$$0.1h_1 = 0.9(h_2 + \alpha'_2 h_f g_2')$$

$$0.1h_1 = 0.9h_2$$

$$\alpha'_2 = 0.96$$

reduction between stages ① stage

stage < gen stage

A convergent, divergent nozzle is supplied with steam at a ps. of 1 MN/m^2 & 225°C supersaturated expansion occurs acc. to $Pv^{1.3} = c$ in the nozzle down to an exit ps. of 0.32 MN/m^2 . the exit dia. of nozzle is 25mm. if the flow through the nozzle is choked

- a) exit velocity b) mass flow rate c) throat diameter

Given data;

$$P_1 = 1 \text{ MN/m}^2 = 1 \times 10^6 \text{ Pa} = 10 \text{ bar}$$

$$T_1 = 225^\circ\text{C}$$

$$Pv^n = c, n = 1.3$$

$$P_2 = 0.32 \text{ MN/m}^2 = 3.2 \text{ bar}$$

$$d_2 = 25 \text{ mm}$$

By using steam table at 10 bar

$$h_f = 762.6 \text{ kJ/kg}, s_f = 2.138 \text{ kJ/kg K}$$

$$h_{fg} = 2013.6 \text{ kJ/kg}, s_{fg} = 4.445 \text{ kJ/kg K}$$

$$t_{sat} = 179.7, v_g = 0.19430 \text{ m}^3/\text{kg}$$

by using steam table at 3.2 bar

$$h_f = 570.9 \text{ kJ/kg}, s_f = 1.695 \text{ kJ/kg K}$$

$$h_{fg} = 2156.7 \text{ kJ/kg}, s_{fg} = 5.274 \text{ kJ/kg K}$$

$$t_{sat} = 135.8^\circ\text{C}, v_g = 0.56995 \text{ m}^3/\text{kg}$$

state ① super heated region

$$T_{sup} > T_{sat}$$

$$h_1 = h_g + c_p (T_{sup} - T_{sat})$$

$$= h_f + h_{fg} + c_p (T_{sup} - T_{sat})$$

$$= 762.6 + 2013.6 + 2.1 (225 - 179.9)$$

$$h_1 = 2870.91 \text{ kJ/kg}$$

The nozzle in the process is isentropic

$$S_1 = S_2$$

$$s_f + s_{fg} + c_p \ln \left(\frac{T_{sup}}{T_{sat}} \right) = s_{f2} + x_2 s_{fg2}$$

$$762.6 + 2013.6 + 1.005 \ln \left(\frac{225}{179.9} \right) = 1.695 + x_2 \times 5.274$$

$$6.8052 = 1.695 + x_2 \times 5.274$$

$$x_2 = 0.96$$

$$h_2 = h_f + x_2 h_{fg}$$

$$= 570.9 + 0.96 \times 2156.7$$

$$= 2641.332 \text{ kJ/kg}$$

$$\therefore V_2 = 44.72 \sqrt{h_1 - h_2}$$

$$= 44.72 \sqrt{2870.91 - 2641.332}$$

$$V_2 = 677.58 \text{ m/s}$$

$$\therefore \text{mass flow rate, } m = \frac{m}{t} = \frac{\rho V}{t} = \rho A \frac{L}{t} = \rho A V = \frac{AV}{\rho}$$

$$V = V_g x = 0.54 \text{ m}^3/\text{kg}$$

$$m = \frac{AV}{\rho} = \frac{\pi (25 \times 10^{-3}) \times 677.58}{0.54}$$

$$m = 0.627 \text{ kg/s}$$

Throat dia;

$$\text{Critical pr. condition}, \frac{P_3}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n-1}{n}}$$

$$P_3 = \left(\frac{2}{1.3+1}\right)^{\frac{1.3-1}{1.3}} \times 10$$

$$P_3 = 5.457 \text{ bar}$$

acc. to mass conservation principle

$$m_1 = m_2 = m_3 = 0.627 \text{ kg/sec}$$

by using steam table at 5.45 bar

$$h_{f3} = 652.8 \text{ kJ/kg}, s_{f3} = 1.890 \text{ kJ/kg K}$$

$$h_{fg3} = 2098.1 \text{ kJ/kg}, s_{fg3} = 4.903 \text{ kJ/kg K}$$

$$t_{sat3} = 154.8^\circ\text{C}, Vg_3 = 0.34844 \text{ m}^3/\text{s}$$

$$h_3 = h_{f3} + x_3 h_{fg3}$$

$$= 652.8 + 1 \times 2098.1$$

$$= 2750.91 \text{ kJ/kg}$$

$$V_3 = 44.72 \sqrt{h_1 - h_3}$$

$$= 44.72 \sqrt{2870.91 - 2750.91}$$

$$V_3 = 189.90 \text{ m/s}$$

$$\dot{m} = \frac{AV}{U}$$

$$0.627 = \frac{\pi d_3^2}{4} \times 489.90$$

$$0.34844$$

$$0.2184 = \frac{\pi d_3^2}{4} \times 489$$

$$0.2184 = 384.76 d_3^2$$

$$d_3 = 0.0238 \text{ m}$$

$$d_3 = 2.38 \text{ cm}$$

Mach number (m):

It is the ratio of velocity of object to the velocity of sound. It is indicated by (m). It has no units.

$$M = \frac{\text{velocity of object}}{\text{velocity of sound}} = \frac{V_o}{c}$$

velocity of light = 3×10^8 m/s

velocity of sound = 330 m/s = 1188 km/hr

$$M = \frac{1200}{1200} = 1 \rightarrow \text{sonic speed}$$

$$M = \frac{1100}{1200} = 0.91 < 1 \rightarrow \text{subsonic}$$

$$M = \frac{1300}{1200} = 1.1 > 1 \rightarrow \text{supersonic}$$

Criteria for selection of nozzle types

Assumptions:

- The flow should be one dimensional.
- The change in area & curvature along the duct or gradual.
- The flow should be steady flow.
- All the mechanical properties are leaving.

Steady flow energy eqⁿ:

$$E_{\text{inlet}} = E_{\text{exit}}$$

$$\rho + P_1 V_1 + U_1 + \frac{1}{2} V_1^2 + g h_1 = \rho + P_2 V_2 + U_2 + \frac{1}{2} V_2^2 + g h_2$$

Nozzle follows the process is isentropic.

$$h = u + PV$$

nozzle follows the process is, $\text{is}(m)$ condition $M = M_0$

$$h_1 + \frac{1}{2}V_1^2 + g h_1' = h_2 + \frac{1}{2}V_2^2 + g h_2' \quad \text{att. ei. to}$$

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 \quad \text{brwse to ptisalev}$$

$$\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2 = h_2 - h_1 \quad \text{brwse to ptisalev}$$

$$d(h, E) = d(\Delta h) \quad \text{brwse to ptisalev} = M$$

1st law of TD $\text{elrn. 801x8} = \text{trfpl. to ptisalev}$

$$dh = du + dw \quad \text{elrn. 828} = \text{brwse to ptisalev}$$

$$h = u + Pv \quad \text{brwse to ptisalev}$$

$$dh = du + Pdv + vdp \quad \text{brwse since } \frac{du}{dh} = \frac{\partial u}{\partial h} = m$$

$$dh = dq + vdp \quad \text{brwse since } \frac{dp}{dh} = \frac{\partial p}{\partial h} = m$$

$$dq = dh - vdp$$

$$dq = dq + vdp + d\left(\frac{c^2}{2}\right)$$

$$-vdp = d\left(\frac{c^2}{2}\right) \quad \text{to mitsalev not matis}$$

$$-vdp = \frac{dc}{2}dc \quad \text{sensitivity}$$

$$cdc = -vdp \quad \text{brwse to mitsalev}$$

$$dc = -\frac{vdp}{c} \quad \text{brwse to mitsalev}$$

considering the mass flow rate (m)

$$m = \frac{AV}{v} \quad \text{project area times density att. 1A}$$

applying ln on b.s

$$\ln(m) = \ln\left(\frac{AV}{v}\right) \quad \text{since } \ln(m) = \ln(A) + \ln(v)$$

$$\ln(m) = \ln A + \ln C - \ln V \quad \text{since } A = \frac{C}{V}$$

applying differentiation

$$\frac{dA}{A} + \frac{dc}{C} - \frac{dv}{v} = d \cdot \ln(m) \quad vq + w = d$$

$$\frac{dA}{A} + \left(\frac{vdP}{c^2} \right) - \frac{dv}{v} = 0 \quad (108 P. 0) \text{ pp. } 0 < m < 1$$

$$\frac{dA}{A} - \frac{vdP}{c^2} - \frac{dv}{v} = 0 \quad \text{or } \frac{dv}{v} = \frac{Ab}{A}$$

$$\frac{dA}{A} = \frac{vdP}{c^2} + \frac{dv}{v} \quad \text{or } \left[\frac{dv}{v} + c \right] = \text{velocity}$$

(triple) $v = \text{sp. vol.}$

$$\frac{dA}{A} = vdP \left(\frac{1}{c^2} + \frac{dv}{v^2 dp} \right) \quad \text{or } \frac{dv}{v} = \frac{Ab}{A}$$

Since eqn. is valid at water ($m < 1$) \rightarrow $\frac{dv}{v} = \frac{Ab}{A}$

$$\frac{dA}{A} = vdP \left[\frac{1}{c^2} + \frac{dv}{v^2 dp} \right] \quad (m < 1) \quad \frac{dv}{v} = \frac{Ab}{A}$$

$$a^2 = -\frac{vdP}{dv} \quad \text{therefore } a \text{ will become constant}$$

$$a = \text{velocity of sound} \quad \rightarrow \text{watt addition (c)}$$

$$\frac{dA}{A} = vdP \left(\frac{1}{c^2} - \frac{1}{a^2} \right)$$

$$\text{mach no.} = \frac{c}{a} = m$$

$$\frac{dA}{A} = \frac{vdP}{c^2} \left[1 - \frac{c^2}{a^2} \right]$$

$$\frac{dA}{A} = \frac{vdP}{c^2} (1-m^2)$$

$$= -\frac{vdP}{c} \left(\frac{1}{c} \right) (-1+m^2)$$

reject between eqn. to make working 0-1

$$\frac{dA}{A} = \frac{dc}{c} (m^2-1) \quad \text{so front working 0-1}$$

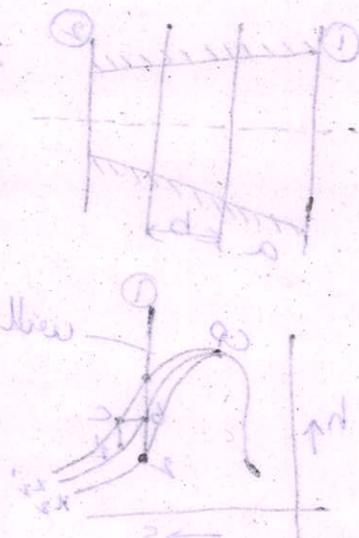
case i - when the fluid is sub-sonic :-

$$\text{mach no.} = \frac{\text{velocity of object}}{\text{velocity of sound}} \quad \boxed{m < 1} = \text{subsonic}$$

$m \geq 1$ = sonic

ent medium - working $m > 1$ = super-sonic

$$\frac{dA}{A} = \frac{dc}{c} (m^2-1)$$



Ex: $m = 0.99$ ($0.9801 - 1$)

$$\frac{dA}{A} = -ve$$

$$dA = (-ve) A = -ve$$

$dA = -ve$ (Convergent)

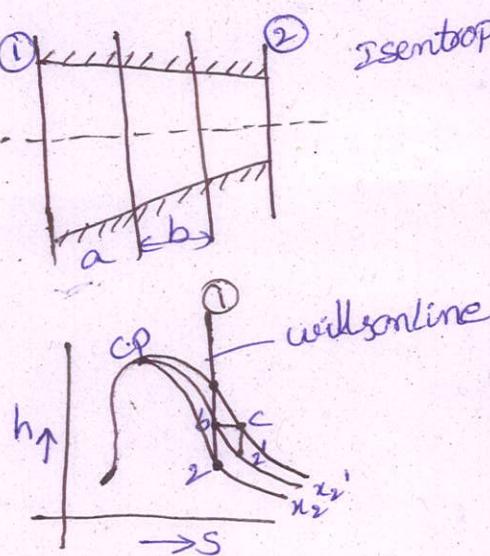
$$A_2 - A_1 = -ve$$

case ii :- ($m > 1$) when the fluid is supersonic

$$\frac{dA}{A} = \frac{dc}{c} (m^2 - 1) \quad \text{divergent.}$$

super-saturated flow / condensate $\xrightarrow{\text{shock}} \text{willson line}$

(or) metastable flow :-



Isentropic process $[S=c]$

$$\left(\frac{1}{P_0} - \frac{1}{P_2} \right) q_{bv} = \frac{Ab}{A}$$

$$m - \frac{2}{5} = \text{am. abm.}$$

$$\left(\frac{1}{P_0} - \frac{1}{P_2} \right) q_{bv} = \frac{Ab}{A}$$

steam steam \xrightarrow{C} sudden condensation

$$(m+1) \left(\frac{1}{P_0} - \frac{1}{P_2} \right) q_{bv} = \frac{Ab}{A}$$

1-a process steam or super heated region

a-b process - acting as steam, velocity $\uparrow b$

b-c process - sudden condensation to form or condensation shock

c-2 process - It expands naturally \rightarrow am. abm.

1-b process - willson line

$$(1-f_m) \frac{Ab}{A} = \frac{Ab}{A}$$

- Effects of super-saturation flow / willson line:-
- velocity of nozzle at exit will decreases.
 - volume flow rate / discharge will increase (2 to 5% ↑)
 - The dryness fraction is also increase as compare with theoretical process min, $n = 0.96$

Degree of under cooling:-
 Difference in the temp. at point 'c' & temp. at point 'B' is known as degree of under cooling.

Degree of super-saturation (or) Super saturation ratio:-
 It is ratio of actual pr. to the saturated pr. corresponding to the point 'c'.

Steam enters at convergent & divergent nozzle at 10 bar & 240°C with velocity 50 m/s. It is discharged at 0.5 bar with a velocity of 978 m/s. The expansion is upto throat & with friction in the divergent part. Determine a) final quality of steam b) exit dia. required if the dia. throat is 10mm. c) no. of nozzles required for a steam flow rate of 60 kg/mm^2 d) % of overall isentropic drop lost in the friction in the divergent part.

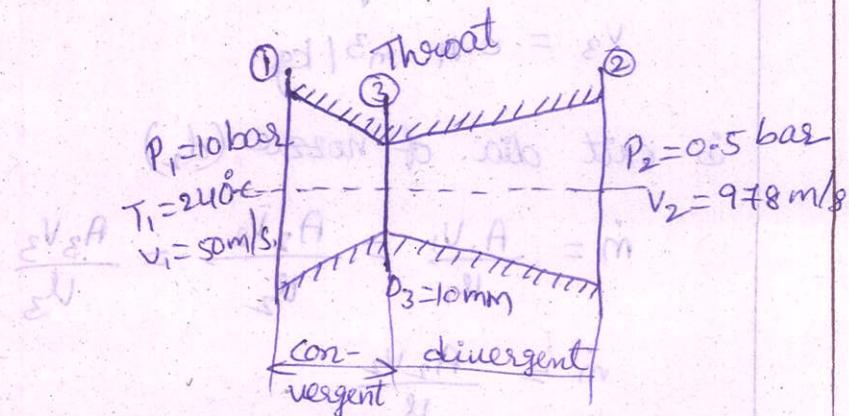
Given data:

$$P_1 = 10 \text{ bar}$$

$$T_1 = 240^{\circ}\text{C} = 513 \text{ K}$$

$$V_1 = 50 \text{ m/s}$$

$$P_2 = 0.5 \text{ bar}$$



$V_2 = 978 \text{ m/s}$ (exit vel), water - goes to nozzle

$$D_3 = 10 \text{ mm} = \frac{\pi}{4} (0.01)^2 = 7.853 \times 10^{-5} \text{ m}^2 \rightarrow \text{flow rate}$$

$$\dot{m}_3 = 60 \text{ kg/min} = \frac{60 \text{ kg}}{60 \text{ sec}} = 1 \text{ kg/sec}$$

a) final quality of steam ($x_2 > 2$)

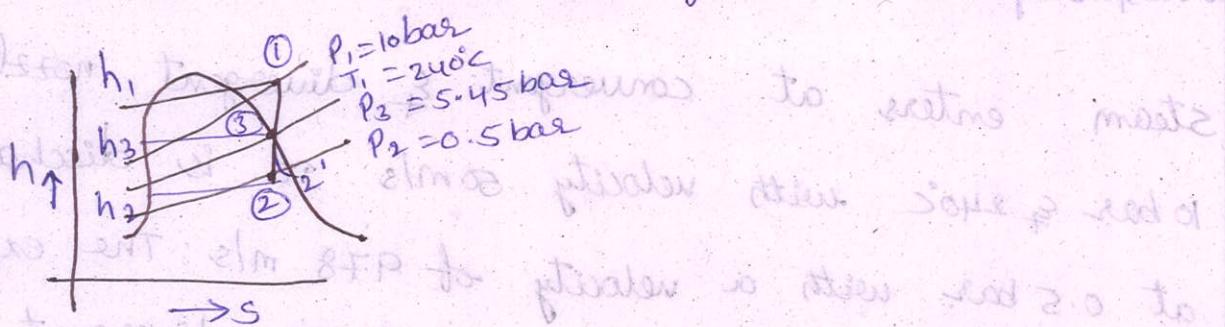
Process involved in nozzle (Isentropic)

By using critical pr. eqn

$$\frac{P_3}{P_1} = \left[\frac{2}{n+1} \right]^{\frac{n}{n-1}}$$

$$P_3 = P_1 \left[\frac{2}{n+1} \right]^{\frac{n}{n-1}}$$

state ① in superheated region



from mollier chart;

$$P_3 = 5.45 \text{ bar}$$

$$h_1 = 2930 \text{ kJ/kg}$$

$$h_2 = 2390 \text{ kJ/kg}$$

$$h_3 = 2770 \text{ kJ/kg}$$

$$V_2 = 3 \text{ m}^3/\text{kg}$$

$$V_3 = 0.1 \text{ m}^3/\text{kg}$$

b) exit dia. of nozzle (d_2)

$$\dot{m} = \frac{A_1 V_1}{U_1} = \frac{A_2 V_2}{U_2} = \frac{A_3 V_3}{U_3}$$

$$\dot{m} = \frac{A_2 V_2}{U_2}$$

Unit-2 Pg-21 P2

consider mass flow rate at throat

$$\dot{m} = \frac{A_3 V_3}{V_3}$$

$$V_3 = 44.72 \sqrt{h_1 - h_3} \quad (= 44.72 \sqrt{2930 - 2770})$$

$$V_3 = 565 \text{ m/s}$$

$$\text{exit dia. of nozzle, } d = \frac{A_1 V_1}{V_3} = \frac{A_2 V_2}{V_3} = \frac{A_3 V_3}{V_3}$$

$$\dot{m} = \frac{A_2 V_2}{V_3}$$

consider mass flow rate of throat 28-FFH

$$\dot{m} = \frac{\pi (0.01)^2 \times 565}{0.4 \times 21.52 \times 10^3} = 0.1109 \text{ kg/sec}$$

$$\text{no. of nozzles} = \frac{\text{total mass flow rate}}{\text{single nozzle mass flow rate}}$$

$$= \frac{28-FF-08P8}{0.1109} = 9.0$$

$$\therefore \text{no. of nozzle} \approx 10$$

$$\dot{m} = \frac{A_2 V_2}{V_2}$$

$$0.11 = \frac{\pi d_2^2 \times 978}{3}$$

$$0.11 \times 3 = \frac{\pi d_2^2 \times 978}{3}$$

$$d_2^2 = \frac{0.11 \times 3}{\pi \times 978}$$

$$d_2^2 = \frac{0.296 \times 10^{-4}}{3}$$

$$d_2 = \sqrt{0.296 \times 10^{-4}}$$

$$d_2 = 0.02072 \text{ m (or) } 20.72 \text{ mm}$$

Unit-2, Pg-23/28

$$V_2 = 44.72 \sqrt{(h_1 - h_3) + (h_3 - h_2')}$$

$$q_{78} = 44.72 \sqrt{(2930 - 2770) + (2770 - h_2')}$$

$$\frac{q_{78}}{44.72} = \sqrt{160 + (2770 - h_2')}$$

$$21.86 = \sqrt{160 + (2770 - h_2')}$$

Sq. on b.s.

$$477.85 = 160 + 2770 - h_2'$$

$$477.85 = 2930 - h_2'$$

$$h_2' = 2930 - 477.85$$

$$h_2' = 2452.15 \text{ kJ/kg}$$

$$\eta_{\text{drop}} = \frac{h_1 - h_2'}{h_1 - h_2} \times 100$$

star wolf steam desuperheating

$$= \frac{2930 - 2452.15}{2930 - 2390} \times 100$$

$$\eta_{\text{used}} = 88.49$$

drop in nozzle is $\approx 11.5\%$

$$8FP \times \frac{\pi D^2}{4} = 11.0$$

$$8FP \times \frac{\pi D^2}{4} = 8 \times 11.0$$

$$\frac{8 \times 11.0}{8FP \times \frac{\pi D^2}{4}} = 11.0$$

$$P_{\text{out}} \times P_{\text{FCN}} = 11.0$$

$$P_{\text{out}} \times P_{\text{FCN}} = 11.0$$

A convergent-divergent nozzle is supplied with steam at a pr. of 1 MN/m^2 & 225°C supersaturated expansion occurs acc. to $PV^{1.3} = C$ in the nozzle exit pr. 0.32 MN/m^2 . the exit dia. of the nozzle is 25mm if the flow through the nozzle is choked. determine a) exit velocity b) mass flow rate c) throat dia.

Given data;

$$P_1 = 1 \text{ MN/m}^2 = 1 \times 10^6 / 10^5 \text{ bar}$$

$$P_1 = 10 \text{ bar}$$

$$T_1 = 225^\circ\text{C}$$

$$P_2 = 3.2 \text{ bar}$$

$$D_2 = 25 \text{ mm}$$

$$n = 1.3$$

max. mass flow rate condition,

$$\frac{P_3}{P_1} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}}$$

$$P_3 = \left(\frac{2}{1.3+1} \right)^{\frac{1.3}{1.3-1}} \times 10 = 0.82 \cdot 0$$

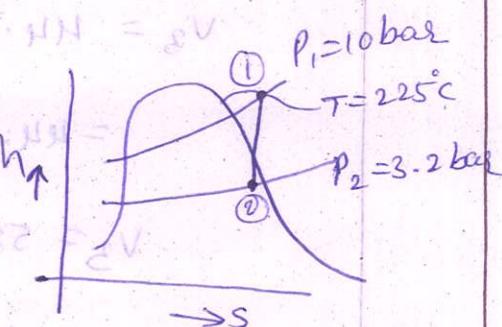
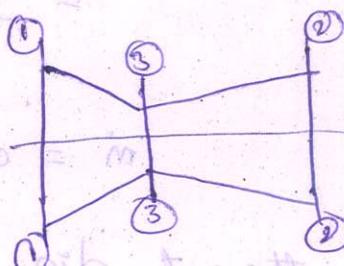
$$P_3 = 5.45 \text{ bar}$$

By using mollier chart

$$h_1 = 2890 \text{ kJ/kg}, v_1 = 0.6$$

$$h_2 = 2680 \text{ kJ/kg}, v_2 = 0.41 \cdot 2$$

$$h_3 = 2750 \text{ kJ/kg}, v_3 = 0.10 \cdot 2$$



exit velocity;

$$V_2 = 44.72 \sqrt{h_1 - h_2} \quad \text{inches of mercury}$$

$$= 44.72 \sqrt{2890 - 2680} \quad \text{ft. of water}$$

$$V_2 = 648.05 \text{ m/s at } 20^\circ\text{C}$$

$$\text{mass flow rate, } \dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} = \frac{A_3 V_3}{v_3}$$

$$\dot{m} = \frac{A_2 V_2}{v_2}$$

$$= \frac{\pi (25 \times 10^3)}{4} \times 648.05$$

$$\dot{m} = 0.530 \text{ kg/sec}$$

throat dia, velocity at throat

$$V_3 = 44.72 \sqrt{(h_1 - h_3)}$$

$$= 44.72 \sqrt{2890 - 2750}$$

$$V_3 = 529.1 \text{ m/s}$$

$$\text{throat dia. } \dot{m} = \frac{A_3 V_3}{v_3}$$

$$\dot{m} = \frac{\pi d_3^2 \cdot V_3}{v_3}$$

$$0.530 = \frac{\pi d_3^2 \times 529.1}{0.4}$$

$$d_3^2 = \frac{0.530 \times 0.4}{\pi \times 529.1}$$

$$d_3^2 = 5.1016 \times 10^{-4}$$

$$d_3 = \sqrt{5.1016 \times 10^{-4}}$$

$$d_3 = 0.0225 \text{ m} \approx 22.58 \text{ mm}$$

1.2 kg/sec of steam enters a nozzle at 12 bar & 250°C expands upto 2 bar. Condensation doesn't occur while the steam is in nozzle. Find the throat area, degree of under cooling.

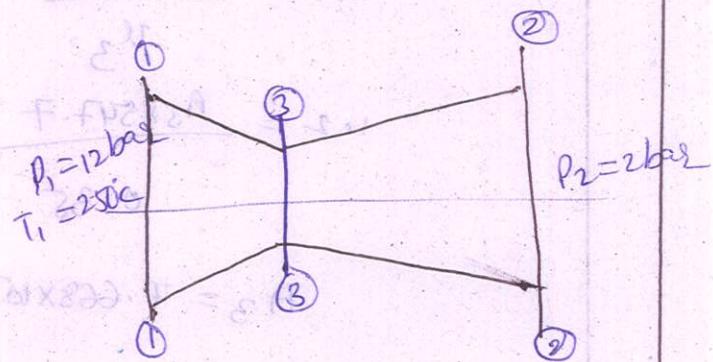
Given data;

$$m = 1.2 \text{ kg/sec}$$

$$P_1 = 12 \text{ bar}$$

$$T_1 = 250^\circ\text{C}$$

$$P_2 = 2 \text{ bar}$$



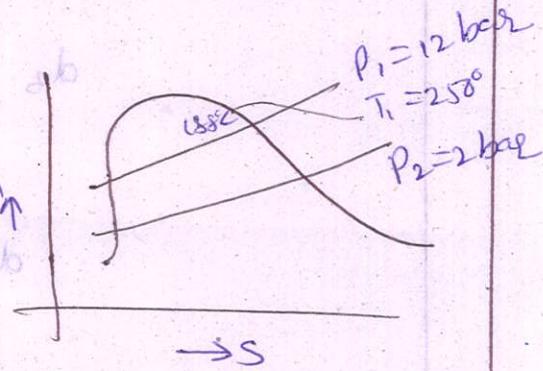
condition for max. discharge

$$\frac{P_3}{P_1} = \left[\frac{2}{n+1} \right]^{\frac{n}{n-1}}$$

$$P_3 = \left[\frac{2}{1.3+1} \right]^{\frac{1.3}{1.3-1}}$$

$$(n=1.3)$$

$$P_3 = 6.54 \text{ bar}$$



By using mollier chart;

$$h_1 = 2940 \text{ kJ/kg}$$

$$h_2 = 2600 \text{ kJ/kg}$$

$$h_3 = 2790 \text{ kJ/kg}$$

$$V_2 = 0.85$$

$$V_3 = 0.35$$

mass flow rate;

$$m = \frac{A_1 V_1}{V_1} = \frac{A_2 V_2}{V_2} = \frac{A_3 V_3}{V_3}$$

velocity at throats meets at 90° for e.i.

$$V_3 = 44.72 \sqrt{h_1 - h_3}$$

$$= 44.72 \sqrt{2940 - 2790} \text{ m/s at throat}$$

$$V_3 = 547.7 \text{ m/s at throat}$$

$$m = \frac{A_3 \times V_3}{u_3}$$

$$1.2 = \frac{A_3 \times 547.7}{0.235}$$

$$A_3 = 7.668 \times 10^{-4}$$

$$\frac{\pi}{4} d_3^2 = 7.668 \times 10^{-4}$$

$$d_3 = \sqrt{\frac{7.668 \times 10^{-4}}{\pi} \times \frac{e_i}{e_i + 0.1}} = \frac{e_i}{\sqrt{1+0.1}}$$

$$d_3 = 0.0312 \text{ m} \quad (or) \quad \left[\frac{e_i}{1+0.1} \right] = 0.9$$

$$d_3 = 31.24 \text{ mm}$$

$$end \rightarrow d = 0.9$$

throats without filter = d

filter off = d

filter off = d

$$28.0 = d$$

$$28.0 = d$$

total wall area

$$\frac{e_i A}{e u} = \frac{e_i A}{e u} = \frac{e_i A}{e u} = \frac{e_i A}{e u} = m$$